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Asymmetrically warped compactifications and gravitational Lorentz violation

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Abstract: In a variant of the Randall-Sundrum braneworld model which has a charged black hole in an anti-de Sitter bulk, the $4D$ speed of gravity depends upon one's location in the bulk, and in general differs from the speed of light on a given 3-brane. We apply phenomenological constraints on the difference in the speeds of light and gravity to models which use the warping to solve the hierarchy problem. In particular, consideration of the gravi-Čerenkov radiation of Kaluza-Klein gravitons by ultra-high energy cosmic rays, following the approach of Moore and Nelson, leads to a stringent limit on the fractional difference between the graviton and photon speeds in RS-like models: it must be less than a part in 10^{23}.

Keywords: Field Theories in Higher Dimensions, D-branes, Black Holes, High Energy Cosmic Rays.

Contents

1. Introduction

The violation of Lorentz symmetry at high energies would be a revolutionary discovery, giving us important clues about the nature of physics beyond the standard model [1]–[4]. It has been argued that such violations are predicted by some theories of quantum gravity [5]. Although string theory does not necessarily predict Lorentz violation, it can do so via background fields which lead to noncommutative geometry [6], or which explicitly violate Lorentz invariance in the gravitational sector [7].

Nevertheless, it is not easy to invent fundamental theories which break Lorentz symmetry in a plausible way. Therefore it is useful to consider new possibilities for breaking Lorentz symmetry. A novel approach was presented by Csaki et al. [8], in the context of the Randall-Sundrum (RS) solution to the hierarchy problem [9], involving two branes separated by an extra dimension with the geometry of $5D$ anti-de Sitter space. Although the original RS model had Lorentz symmetry, ref. [8] (hereafter called CEG) considered a variant in which the $5D$ bulk contains a black hole of charge Q and mass μ, giving a metric (the AdS Reissner-Nördstrom black hole solution) with the line element

$$ds^2 = -h(r)\,dt^2 + \frac{r^2}{l^2}d\vec{x}^{\,2} + \frac{1}{h(r)}dr^2\,; \tag{1.1}$$

$$h(r) = \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4}\,. \tag{1.2}$$

The extra dimension has coordinate r, and the length scale l characterizes the curvature of the AdS geometry, which is determined by the negative bulk cosmological constant through $l^2 = -6/\Lambda$. (We work in units where the $5D$ gravitational constant $\kappa_5 = 1$.)

The presence of a bulk black hole (BH) can be physically motivated: it was argued in [10] that emission of gravitational radiation from a brane in the early universe can lead to the formation of a BH in the bulk. Possible cosmological consequences of gravity traveling faster than light in this scenario were considered in references [11]. However in the cosmological solutions, the brane moves away from the BH as the universe expands, and as a result the presence of the BH quickly becomes irrelevant for the brane observer: its effects are redshifted away. The presence of charge on the BH is motivated by the desire to have a solution in which the brane-BH distance remains constant, and therefore its effects can also be important at late times.

Along any $4D$ slice of constant r, the metric (1.1) is Lorentz invariant, with speed of light given by $dx/dt = (l/r)h^{1/2}$. Thus an observer on a D-brane which is restricted to a particular value of r does not see direct evidence of Lorentz violation in the matter sector. However gravity propagates in all dimensions, not just along the brane. A $4D$ observer would find that gravity propagates with a different speed than matter.

It might be thought that the weakness of gravitational interactions would render this mild Lorentz violation phenomenologically harmless; after all, the speed of gravity has not been measured. However, there are ways in which it would be manifested. One possibility is that self-energy diagrams with graviton loops would induce speed differences between observable particles, like photons and electrons (where by "speed" we mean the maximum speed, as the momentum becomes infinite). These kinds of constraints were considered in [12].

A more powerful constraint exists in the case where gravity travels slower than visible particles, especially protons [13]. The emission of gravitational Čerenkov radiation would very efficiently damp ultra high-energy cosmic rays (UHECR's) to levels below those which are observed. One of the main observations of the present paper is that this is precisely the situation in asymmetrically warped models in which we are assumed to be living on a negative tension brane, where the weak scale hierarchy can be solved by the warping. In CEG it was assumed that we live on a positive tension brane; in that the speed of gravity is faster than that of light, so that the present strong constraint is not relevant.

In CEG, the asymmetric effects of warping were treated perturbatively. In the present work we also extend their analysis by considering these effects in the nonperturbative regime, and for the Kaluza-Klein (KK) excitations of the graviton. These could be relevant sources of Lorentz violation in colliders, since in RS-like models the KK gravitons couple strongly to the standard model particles [14].

We begin by reviewing the CEG model and their perturbative derivation of the speed of gravity, and we enumerate the most stringent experimental constraints on Lorentz violation in these models. In section 3 we point out that the perturbative treatment can be misleading if the observer is located on the positive tension brane, whereas it gives reliable predictions for observers on the negative tension brane. In section 4 we show that the KK excitations of the graviton have similar Lorentz-violating properties to the zero mode, considerably strengthening the Čerenkov bound introduced in section 2.

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2. The models; perturbative speed of gravity

2.1 Model with two branes

Similarly to the RS model, CEG cuts the bulk at two positions, call them r_+ and r_-, by the insertion of branes and the use of Z_2 orbifold symmetry to define the discontinuity in the derivatives of the metric functions at the branes. The parts of the solution $r > r_+$ and $r < r_-$ are discarded, and replaced by mirror copies of the kept part of the solution to create a solution with orbifold fixed points at $r_\pm$. Unlike RS, it is not possible to use conventional branes with equation of state $p = -\rho$ at both positions. There are four junction conditions which relate the energy densities $\rho_\pm$ and equations of state $p_\pm = w_\pm \rho_\pm$ to the black hole parameters:

$$\mu\, l^2 = 3\left(1+\frac{w_+}{36}\rho_+^2 l^2\right) r_+^4 = 3\left(1+\frac{w_-}{36}\rho_-^2 l^2\right) r_-^4\,; \tag{2.1}$$

$$Q^2\, l^2 = 2\left(1+\frac{1+3w_+}{72}\rho_+^2 l^2\right) r_+^6 = 2\left(1+\frac{1+3w_-}{72}\rho_-^2 l^2\right) r_-^6\,. \tag{2.2}$$

It is easy to see that taking $w_+ = w_- = -1$ implies that $r_+ = r_-$. Let us consider the physically interesting case of a large hierarchy, where

$$\epsilon \equiv \frac{r_-}{r_+} \sim 10^{-16}\,. \tag{2.3}$$

Physical masses on the negative tension brane at r_- (the "TeV brane") are suppressed relative to their bare values by the factor ϵ, giving a resolution of the weak scale hierarchy problem [9]. By solving the jump conditions (2.1) and (2.2) for ϵ, we see that there are two ways of achieving a small value of ϵ: either by (1) tuning the numerators $(1+\frac{w_+}{36}\rho_+^2 l^2)$, $(1+\frac{1+3w_+}{72}\rho_+^2 l^2)$ to be small, or (2) tuning the denominators $(1+\frac{w_-}{36}\rho_-^2 l^2)$, $(1+\frac{1+3w_-}{72}\rho_-^2 l^2)$ to be large.

1. In the first case, $l\rho_-/6$ and $w_-(l\rho_-/6)^2$ can naturally be of order unity, while w_+ and ρ_+ must be tuned to the values

$$w_+ \cong -1 - 2\epsilon^4\left(1+\frac{w_-}{36}\rho_-^2 l^2\right) < -1\,; \qquad l\rho_+ = 6 + O(\epsilon^4)\,, \tag{2.4}$$

implying the relation

$$\mu = c_1 \frac{Q^2}{r_-^2}\,; \qquad c_1 = \frac{3\left(1+\frac{w_-}{36}\rho_-^2 l^2\right)}{2\left(1+\frac{1+3w_-}{72}\rho_-^2 l^2\right)} = O(1)\,. \tag{2.5}$$

The inequality in (2.4) is a violation of the weak energy condition (albeit a small one), which states that $p \geq -\rho$, and would thus require some exotic kind of stress energy on the Planck brane.

2. In the second case, we need

$$w_- = O\left(\epsilon^2\right); \qquad l\rho_- = O\left(\epsilon^{-6}\right), \tag{2.6}$$

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which implies

$$\mu = c_2 \frac{Q^2}{r_+^2}\,; \qquad c_2 = \frac{3\left(1+\frac{w_\pm}{36}\rho_+^2 l^2\right)}{2\left(1+\frac{1+3w_+}{72}\rho_+^2 l^2\right)} = O(1)\,. \tag{2.7}$$

The relations (2.5), (2.7) are useful for computing the speed of electromagnetic radiation (or any relativistic particles) on the branes. Working to first order in the Lorentz violating parameters μ and Q^2, the deviation in the speed relative to unity is

$$\delta c_{\rm em} = -\frac{\mu\, l^2}{2r_\pm^4} + \frac{Q^2 l^2}{2r_\pm^6} = \frac{l^2\, Q^2}{2r_\pm^4}\left(\frac{1}{r_\pm^2} - \frac{c_i}{r_i^2}\right), \tag{2.8}$$

where the choice $\pm$ refers to which brane one is on, and the label i refers to the choice of tunings immediately above; hence $r_1 = r_-$ and $r_2 = r_+$.

2.2 Model with one brane and a horizon

The preceding discussion assumed the existence of two branes bounding the extra dimensional space. It is also possible to eliminate the TeV brane altogether by not cutting out the small r region containing the black hole. In that case one should choose μ and Q^2 so that $h(r)$ vanishes at least once between $r = 0$ and $r = r_+$. This insures there is a horizon shielding the black hole, in accordance with the cosmic censorship hypothesis. In this scenario we would necessarily be living on the Planck brane at $r = r_+$. The parts of the jump conditions (2.1) and (2.2) involving ρ_+ and w_+ would still be valid. CEG showed that it is not possible to embed the brane and still have a horizon shielding the black hole unless the weak energy condition is again violated, $w_+ < -1$. This is most easily studied when the special relation $4\mu^3 l^2 = 27Q^4$ holds; this is the limiting case where the two horizons of the AdS Reissner-Nördstrom black hole degenerate into a single horizon, located at $r_h/l = (2Q/l^2)^{1/3}/\sqrt{2}$. One can solve the jump conditions to find that the brane's equation of state depends on its position, r_+, and is given by

$$w_+ = -1 - 2\left(\frac{r_h}{r_+}\right)^4 + O\left(\frac{r_h}{r_+}\right)^6. \tag{2.9}$$

Hence the violation of the weak energy condition becomes small as the brane is moved farther away from the black hole. Ref. [15] showed that this violation could be moved into the bulk stress-energy tensor, but never eliminated, so long as the horizon exists. The deviation in the speed of light on the brane in this scenario is given by

$$\delta c_{\rm em} = -\frac{3}{2}\left(\frac{r_h}{r_+}\right)^4 + \left(\frac{r_h}{r_+}\right)^6. \tag{2.10}$$

We mention this case for completeness, but we will not further consider how gravity propagates in this case.

2.3 The speed of gravity; experimental constraint

Now we turn to the speed of gravity. Its determination is simplified by CEG's observation that the dynamics of gravity are the same as those for a massless bulk scalar field, with action and equation of motion

$$S = \frac{1}{2}\int d^4x\, dr \sqrt{-g} g^{AB}\partial_A\phi\partial_B\phi\,; \tag{2.11}$$

$$\frac{\delta S}{\delta\phi} = -\partial_A\left(\sqrt{-g}g^{AB}\partial_B\phi\right) = 0\,. \tag{2.12}$$

The problem can be further simplified by splitting the metric into a pure AdS part and the perturbation involving the black hole parameters. In other words, one works perturbatively to first order in μ and Q^2. The solution for the nth KK mode is separable, $\phi_n(x^\mu, r) = e^{i(\omega_n t - \vec{q}\cdot\vec{x})}\phi_n(r)$, where $\phi_n(r)$ satisfies Neumann boundary conditions at the branes, since the jump conditions (2.1) and (2.2) have already been satisfied by the background solution. The dispersion relation of the graviton zero mode solution in the tower of KK excitations can be determined analytically (in the model with two branes) for a solution with 3-momentum $\vec{q}$:

$$\omega_0 = |\vec{q}| + \delta\omega_0; \quad \delta\omega_0 = \frac{|\vec{q}|\, l^2}{4\,\epsilon^2}\left(-2\frac{\mu}{r_+^4} + \frac{Q^2}{r_+^6}\left(1+\epsilon^{-2}\right)\right). \tag{2.13}$$

The deviation in the speed of gravity from 1 is given by

$$\delta c_{\rm grav} = \frac{\partial\delta\omega_0}{\partial|\vec{q}|}\,. \tag{2.14}$$

The relevant Lorentz-violating observable is the difference $\delta c_{\rm em} - \delta c_{\rm grav}$. In general, the sign of this difference depends on the value of the $O(1)$ constant c_1 or c_2. However for TeV brane observers, if the hierarchy is very large ($\epsilon \ll 1$), and if we exlude the case (2.4) and (2.5) which violates the weak energy condition, the difference is completely dominated by the term $l^2Q^2/(2r_-^6)$ in (2.8), and leads to

$$\delta c_{\rm em} - \delta c_{\rm grav} = \frac{l^2Q^2}{2r_-^6}\,. \tag{2.15}$$

Because the difference is positive, the stringent constraints due to gravitational Čerenkov radiation of UHECR protons is applicable [13]:

$$\delta c_{\rm em} - \delta c_{\rm grav} < 2\times 10^{-15}\ (10^{-19})\,, \tag{2.16}$$

where the less stringent limit assumes the UHECR's are of galactic origin, and the more stringent one applies if they originally come from neutrinos originating from cosmological distances. Therefore we have the bound

$$\frac{\delta h(r_-)}{h(r_-)} \cong \frac{l^2Q^2}{r_-^6} \lesssim 4\times 10^{-15}\ (10^{-19})\,. \tag{2.17}$$

Notice that the quantity on the left is the fractional perturbation to the metric function $h(r)$ due to the dominant Lorentz violating term, evaluated on the TeV brane: $h(r) \cong (r/l)^2(1+\delta h/h)$ It is therefore natural that this is the combination which is experimentally bounded. (2.17) is one of the new constraints derived in this paper.

If we are willing to entertain the possibility of case 1, with exotic matter on the Planck brane which violates the weak energy condition, then the difference of speeds as measured on the TeV brane becomes dependent on the details of the TeV brane stress energy,

$$\delta c_{\rm em} - \delta c_{\rm grav} = \frac{l^2 Q^2}{2r_-^6}(1-c_1)\,. \tag{2.18}$$

If $c_1 < 1$, the Čerenkov bound again applies, in the obvious way. If $c_1 > 1$, gravitational Čerenkov radiation cannot be produced. Then the strongest bounds come from tests for deviations from general relativity in the parametrized post-newtonian (PPN) formalism. A difference between the speed of light and that of gravity implies the existence of a preferred reference frame, the one in which we have been working, where the speeds of gravity and light are independent of direction. In a boosted frame, one of these would become anisotropic, depending upon whether the boost respects Lorentz invariance of the gravitational or the electromagnetic propagation. (One but not both can be preserved, due to the difference in speeds.) In such a situation, the parameter α_2 of the PPN formalism is nonzero. Essentially, we have two metrics, one for photons and one for gravitons, so the effective theory is Rosen's bimetric one [16]. Such theories predict a torque which would cause precession of the sun's spin axis. The latter is closely aligned with the solar system's planetary angular momentum vector. If we assume that the alignment is not coincidental, then precession due to PPN effects is constrained, and leads to the bound [17]

$$|\alpha_2| = 2\frac{|\delta c_{\rm em} - \delta c_{\rm grav}|}{c_{\rm em}} < 1.2\times 10^{-7}\,, \tag{2.19}$$

which carries over to the quantity $\frac{l^2Q^2}{r_-^6}|1-c_1|$.[1]

3. Nonperturbative analysis

The above discussion assumed that the dispersion relation of the graviton gets modified in the simple way which was predicted by treating the Lorentz-violating part of the metric, δh, to first order in perturbation theory. However, at large graviton momenta, the perturbative prediction can be modified. To see this, one must examine the equation of motion (2.12). Let us rewrite it to first order in δh, with $\phi_0^{(0)}$ and $\phi_0^{(1)}$ respectively denoting the zeroth and first order solutions, in powers of δh, for the Kaluza-Klein zero mode. We will also change coordinates to the form $r/r_+ = e^{-ky}$ with $k = l^{-1}$. The wave equation for the zero

[1]We thank Guy Moore for discussions on this point.

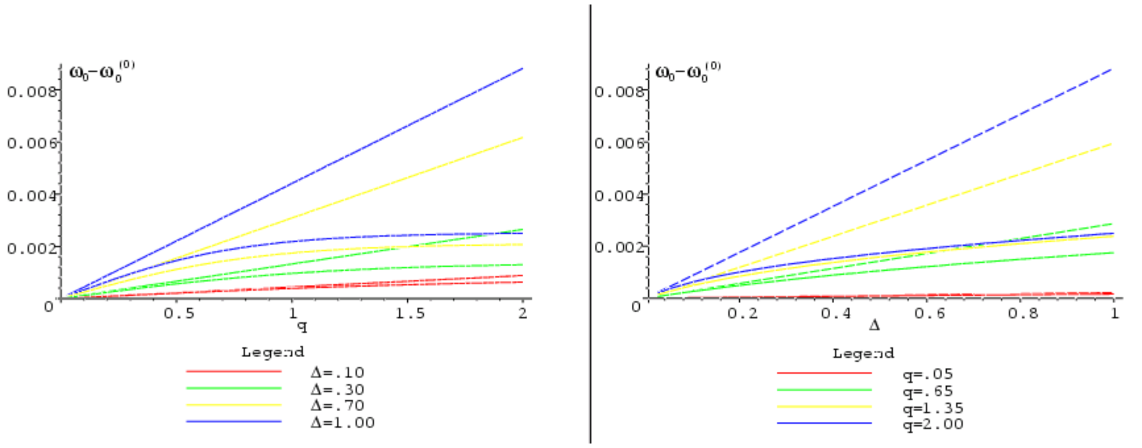

Figure 1: Correction to $\omega_0^{(0)}/k$ as a function of q/k (left) and Δ (right). The upper (straight) lines show the perturbative prediction (2.13), while the lower (curved) lines show the numerical result. Each color represents a different perturbation or momentum, and $y_- = 2/k$.

mode with 3-momentum q is

$$\begin{aligned} \partial_y^2 \phi_0^{(1)}(y) - 4k \,\mathrm{sgn}(y)\, \partial_y \phi_0^{(1)}(y) &= e^{2k|y|} \left(-2q\, \omega_0^{(1)} + q^2 \frac{\delta h}{h}(y) \right) \phi_0^{(0)} \\ &= e^{2k|y|} \left(-2q\, \omega_0^{(1)} + \frac{q^2}{k^2} \left(-\frac{\mu}{r_+^4} e^{4k|y|} + \frac{Q^2}{r_+^6} e^{6k|y|} \right) \right) \phi_0^{(0)} \end{aligned} \tag{3.1}$$

Regardless of how small $\delta h/h$ is, at sufficiently large momenta the source term $q^2(\delta h/h)\,\phi^{(0)}$ becomes so large that it can no longer be reliably treated as a perturbation.

To explore this, we have numerically solved the full, unperturbed graviton wave equation to find the dispersion relation for the zero mode, $\omega_0(q)$, and compared this result to the perturbative prediction (2.13). To make the problem tractable, we have chosen definite values for the brane stress energy parameters. For the tuning of parameters, we adopt case 2 above, eqs. (2.6) and (2.7), so that c_2 depends on the positive tension brane parameters. These we take to be $w_+ = -1$, while leaving ρ_+ free to vary. The full wave equation can be written as

$$\phi''(y) - 4k \,\mathrm{sgn}(y) \left(\frac{1 - \delta\, e^{6k|y|}}{\hat{h}(y)} \right) \phi'(y) + e^{2k|y|} \left(\frac{\omega^2}{\hat{h}^2(y)} - \frac{q^2}{\hat{h}(y)} \right) \phi(y) = 0\,, \tag{3.2}$$

where

$$\hat{h}(y) = 1 + \delta\, e^{4k|y|} \left(-3 + 2e^{2k|y|} \right); \qquad \delta \equiv 1 - \frac{\rho_+^2 l^2}{36}\,. \tag{3.3}$$

In this form, it is clear that the Lorentz-violating effects, coming from $\hat{h} - 1$, are maximized at the TeV brane, where $y = y_-$. For the purposes of comparing exact results to the perturbative ones, we quantify the perturbation by the parameter

$$\Delta \equiv 2\delta e^{6ky_-}\,. \tag{3.4}$$

We expect the perturbative treatment to be valid when $\Delta \ll 1$.

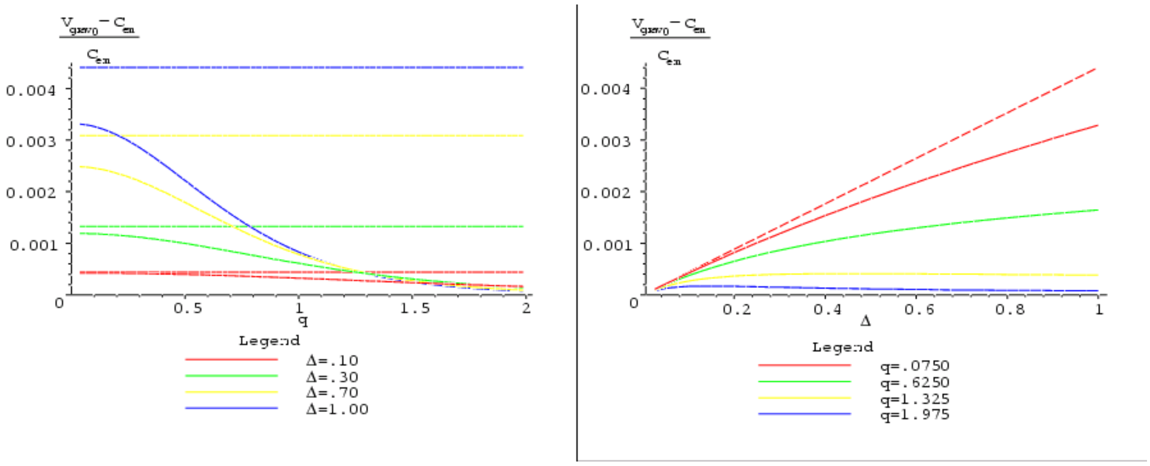

Figure 2: Relative speed difference between the gravity zero mode and a photon on the *positive tension brane* as a function of q (left) and Δ (right) when the physical brane has positive tension. The upper (straight) lines show the perturbative prediction derived from (2.14), while the lower (curved) lines show the numerical result. Each color represents a different perturbation or momentum, and $y_- = 2/k$. In the right panel, all the perturbative curves coincide with each other.

With these definitions we can now compare the exact numerical results with the perturbative ones. The deviation of the dispersion relation of the zero mode, relative to its Lorentz-conserving value, is plotted as a function of momentum for several values of Δ, and as a function of Δ for several momenta, in figure 1, where $\delta\omega$ and q are given in units of $k = l^{-1}$. We have taken a very small hierarchy, $ky_- = 2$ for this illustration, and will comment on the extrapolation to larger values below. The interesting feature is that the true $\delta\omega(q)$ flattens to a constant value at large q, contrary to the perturbative expectation.

If we compare the speed of gravity to that of hypothetical photons which are trapped on the *positive tension brane*, a consequence of this behavior of the graviton dispersion relation is that the speed difference (with speed defined using the group velocity $v = d\omega/dq$) does not remain constant at large momenta, but rather vanishes as $q \to \infty$, as shown in figure 2. At large momenta, gravitons tend toward the same speed as radiation on the Planck brane. This can be understood as follows. The graviton zero mode is localized on the Planck brane. At large q^2, the effect of the terms $(\frac{\omega^2}{\hat{h}^2(y)} - \frac{q^2}{\hat{h}(y)})$ in the equation of motion (3.2) is to localize it even more, thus driving the graviton to resemble more closely radiation which is trapped on the Planck brane. This trend becomes evident for momenta $q \gtrsim 1/l$, where the speed difference is significantly reduced relative to its maximum value at $q = 0$.

As the hierarchy between the Planck brane and the negative tension brane is increased by letting y_- go to larger values, the Lorentz violating effects seen by a Planck brane observer are suppressed, if the parameter Δ is held fixed. The magnitude of $c_{\rm grav} - c_{\rm em}$ scales like e^{-2ky_-}. This can be understood analytically, using the perturbative results of the previous section. One can show that, for the present choice of parameters,

$$c_{\rm grav} - c_{\rm em} = \frac{1}{2}\left(\cosh(2ky_-) - 1\right) e^{-4ky_-}\Delta \sim +O\left(e^{-2ky_-}\right). \qquad (3.5)$$

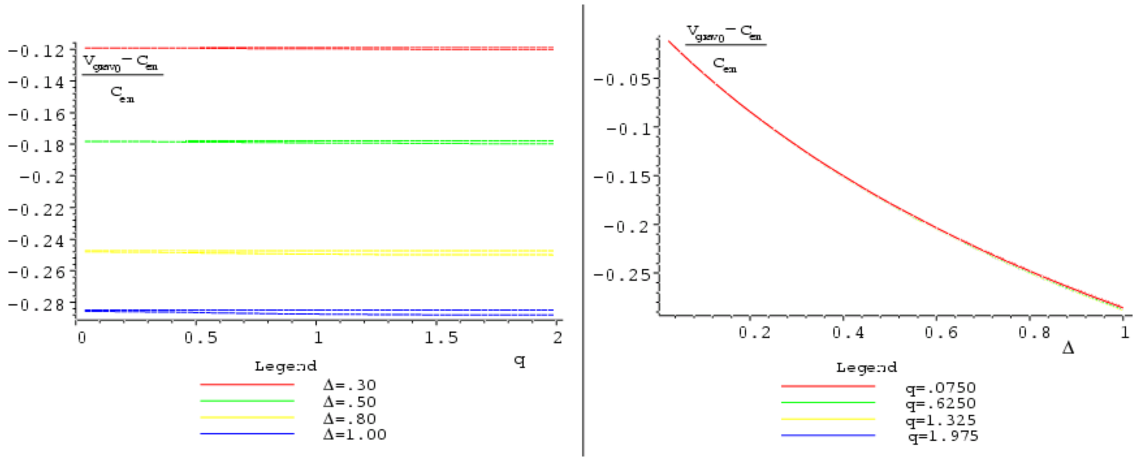

Figure 3: Same as figure 2, but here the photon is on the negative tension (TeV) brane. All curves are essentially coincident in the right panel.

At the same time, the q-axis in figure 2 gets rescaled by a factor of e^{-ky_-}, such that the range of momenta with an appreciable deviation in $c_{\rm grav} - c_{\rm em}$ gets shifted to smaller physical values. In the hierarchy-solving case $e^{-ky_-} \sim 10^{16}$, this corresponds to the TeV scale.

On the other hand, an observer on the TeV brane will not be concerned with this small momentum-dependence in the graviton speed, because the difference between $c_{\rm grav}$ and $c_{\rm em}$ remains relatively large even at high momenta, as shown in figure 3. Moreover, in contrast to the case of the Planck brane observer, the difference $c_{\rm grav} - c_{\rm em}$ remains constant as the hierarchy is increased, if Δ is held fixed. Again, this can be understood by computing the speed difference perturbatively, for the present choice of parameters,

$$c_{\rm grav} - c_{\rm em} = -(\cosh(2ky_-) - 1)e^{-2ky_-}\Delta \sim -O(1) \times \Delta\,. \tag{3.6}$$

The conclusion of this section's analysis is that we can trust the results of the perturbative treatment if we assume that observers are living on the negative tension brane, as one would expect if the hierarchy problem is being addressed. Only for Planck brane observers would it be important to distinguish the perturbative from the exact results.

4. Lorentz-violating KK modes

It would be interesting if Lorentz violating kinematics could be observed in the laboratory, in particle collider experiments. In section 2 we noted that the speed difference between gravity and light on the TeV brane should be less than a few parts in 10^7. Such a small effect would probably be impossible to see at the LHC, even though KK gravitons interact strongly (with TeV-suppressed instead of Planck-suppressed couplings) with standard model particles and would be copiously produced, if sufficiently light.

Even if their Lorentz-violating properties cannot be directly detected, an interesting indirect effect is the Čerenkov emission of KK gravitons from UHECR's, which would strengthen the bound mentioned in section 2. That bound conservatively counted only the damping due to emission of massless gravitons. But KK modes can also be emitted, so

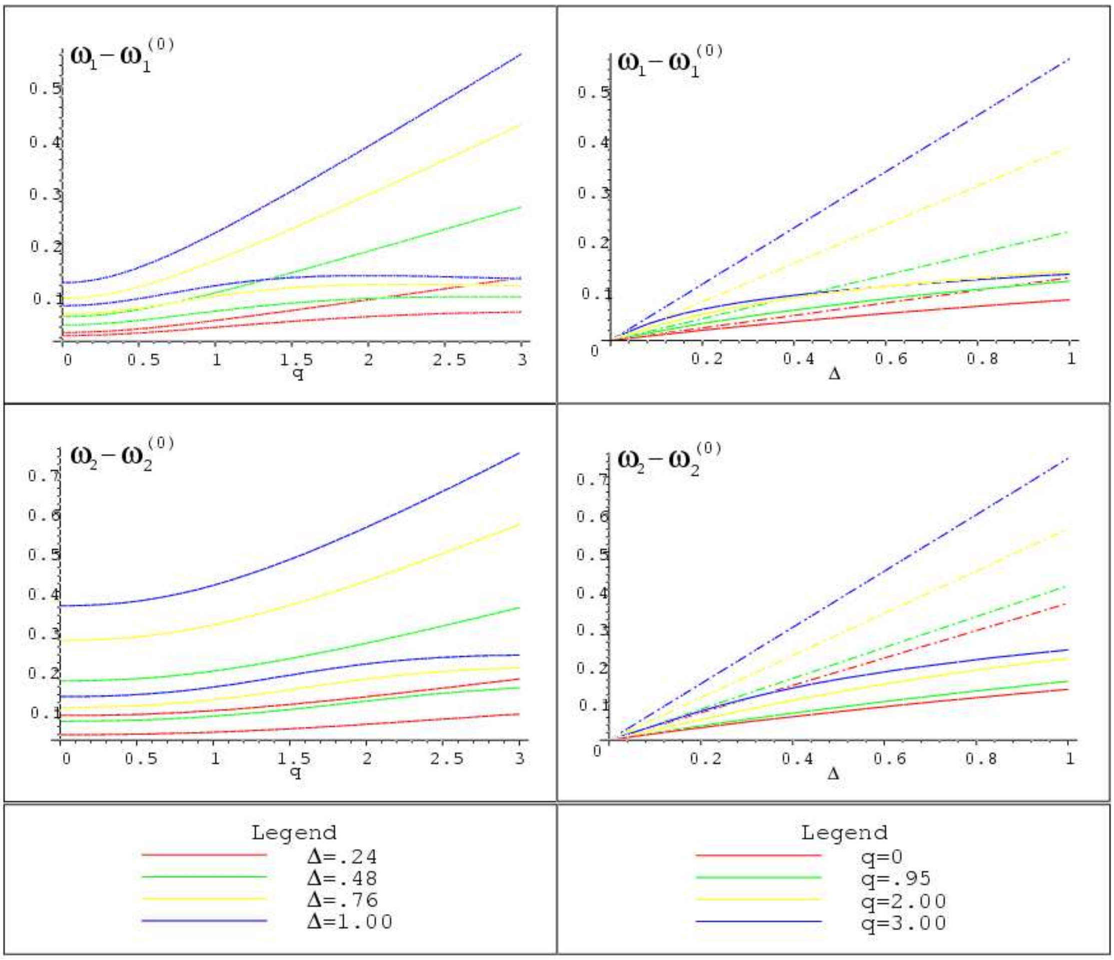

Figure 4: Same as figure 1, but for the first two massive KK modes. The perturbative prediction comes from eq. (A.5).

long as their mass obeys the inequality

$$\frac{m_{\rm KK}^2}{2\vec{p}^{\,2}} < c_{\rm em} - c_{\rm grav}(p)\,, \tag{4.1}$$

where $\vec{p}$ is the momentum of the UHECR. This is the condition for Čerenkov emission to be kinematically allowed. In the situation where the warp factor $\epsilon = 10^{-16}$, the KK mass gap is of order TeV. The highest energy cosmic ray which has been detected had energy 3×10^{11} GeV [18]; thus for a speed difference that saturates the solar system bound (2.19), the number of relevant KK modes is of order 10^5. It is therefore worth exploring whether the KK modes have similar Lorentz-violating kinematics as the graviton zero mode.

We have addressed this problem both numerically and analytically. The analytic approach is to solve the wave equation once again treating δh to first order in perturbation theory, but now expanding around the zeroth order Lorentz-conserving KK wave function, $\phi_n(y) \cong \phi_n^{(0)}(y) + \phi_n^{(1)}(y)$, in order to find the corresponding energy eigenvalue, $\omega_n(q) \cong \omega_n^{(0)}(q) + \omega_n^{(1)}(q)$. The solution is described in detail in the appendix. We have also

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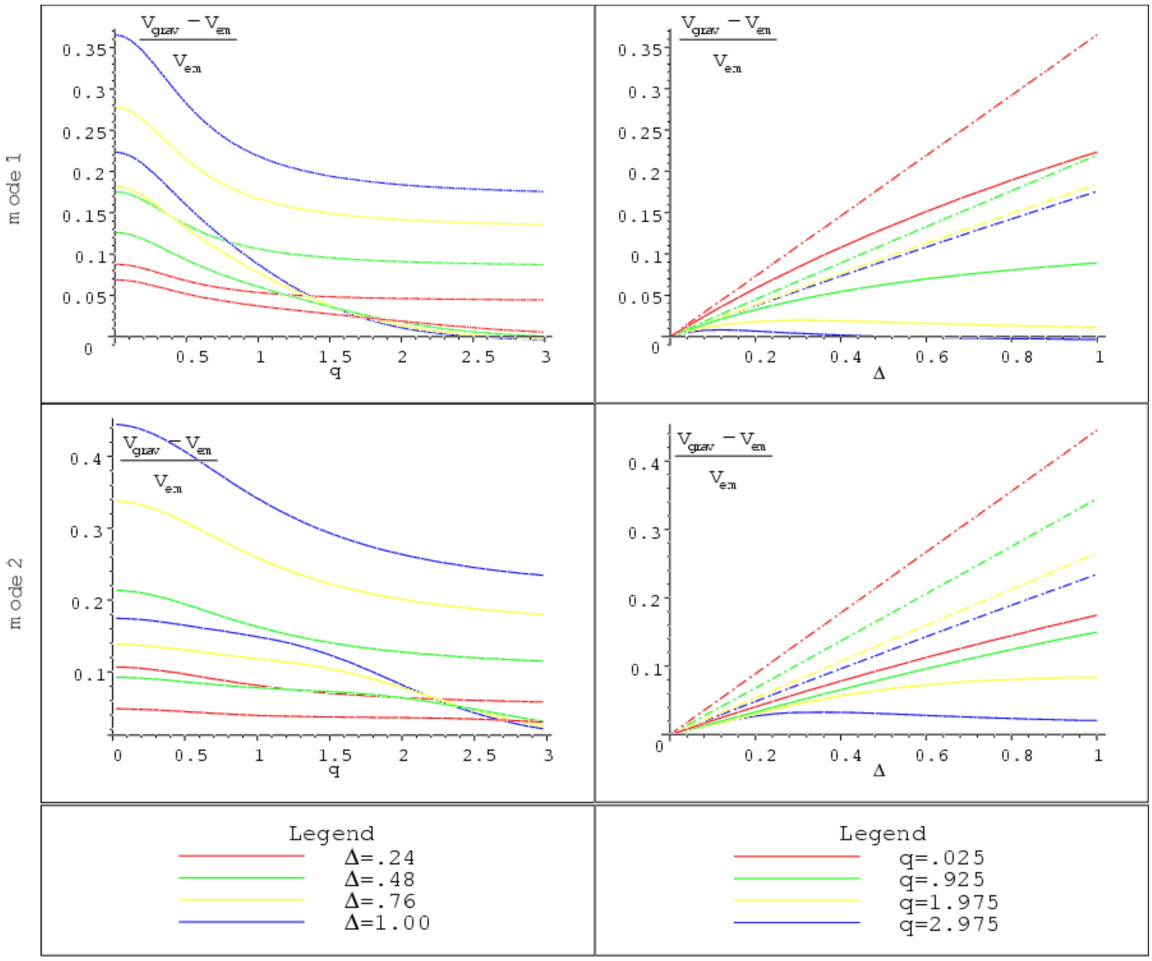

Figure 5: Relative speed difference between the first two massive gravity KK modes and hypothetical particles of the same masses, living on the positive tension brane, as a function of q (left) and Δ (right). Upper curves are perturbative, lower are numerical results.

numerically solved the full wave equation using the shooting method, to find the dispersion relation $\omega_n(q)$. The deviation of $\omega_n(q)$ from the standard result $\omega = q$ is shown for the first two KK modes in figure 4. The comparison between the perturbative and numerical results is shown there.

As we did for the KK zero mode, we can compute the difference in the speed of gravity relative to that of particles trapped on the positive tension brane. In order to highlight the differences which are due to Lorentz violation, we take the conventional trapped particles to have the same rest mass as that of the KK graviton to which it is being compared. The results, shown in figure 5 are qualitatively similar to those for the zero mode: gravity is faster than particles on the Planck brane, but the speed difference becomes smaller at higher momenta.

More interesting is the speed difference relative to particles on the TeV brane. Figure 6 shows that, again like the zero mode, KK gravitons are slower than same-mass particles on the TeV brane. Therefore our expectation that they are produced in gravi-Čerenkov radiation by particles with momenta satisfying (4.1) is justified, and we should revise the bound. Adapting the result of [13] to the tower of TeV-mass-gap gravitons whose couplings are only TeV-suppressed, the rate of energy loss of a UHECR with momentum p and mass m_p is given by

$$\frac{dE}{dt} \sim \frac{p^2}{\mathrm{TeV}^3} \int_0^{q_{\max}} dq\, q \int_0^{M_{\max}} dM\, \sin^4\theta\,, \tag{4.2}$$

where q is the momentum of the emitted KK graviton, M is its mass, and θ is the angle of emission of the Čerenkov radiation. The kinematic limits and θ (in the limit $\theta \ll 1$) are given by

$$\begin{aligned} \theta^2 &= \left(1 - \frac{q}{p}\right) q^{-2}(M_{\max}^2 - M^2) \\ M_{\max}^2 &= q^2 \left(2\,\delta c - \frac{m_p^2}{p^2(1-q/p)}\right) \\ q_{\max} &= p\left(1 - \frac{m_p^2}{2p^2\,\delta c}\right). \end{aligned} \tag{4.3}$$

Here $\delta c = c_{\mathrm{em}} - c_{\mathrm{grav}}$, which must be positive in order for Čerenkov emission to occur. Evaluating the integrals gives

$$\frac{dE}{dt} \sim 0.1\,(\delta c)^{5/2} f(\delta c)\,\frac{p^5}{\mathrm{TeV}^3}\,, \tag{4.4}$$

where the function f is approximately 1 for $\delta c \gg \frac{m_p^2}{2p^2}$ and drops very rapidly to zero (like $(1 - m_p^2/2p^2\delta c)^{11/2}$) as $\delta c \to \frac{m_p^2}{2p^2}$. The bound on δc comes from demanding that dE/dt be less than p/L for a cosmic ray which is propagating over a distance L. For the highest energy cosmic ray observed, which is identified with a proton, the bound is satisfied by taking δc to be below the kinematic limit where $f = 0$. This implies that

$$\delta c < \frac{m_p^2}{2p^2} \cong 10^{-23}\,. \tag{4.5}$$

5. Conclusions

Asymmetric warping can provide a plausible means of introducing Lorentz violation into a theory with extra dimensions, which is essentially a form of spontaneous breaking due to the gravitational background. Since at tree level the violation is confined to the gravitational sector, the effects can be sufficiently weak to be at the borderline of detection. In this paper we have explored some of the consequences of a theory where the asymmetric warping comes from the presence of a charged black hole in the extra dimension. Such a scenario can be compatible with the RS solution to the weak scale hierarchy problem. Unlike RS, in the black hole case it is necessary to allow for nonstandard equa-

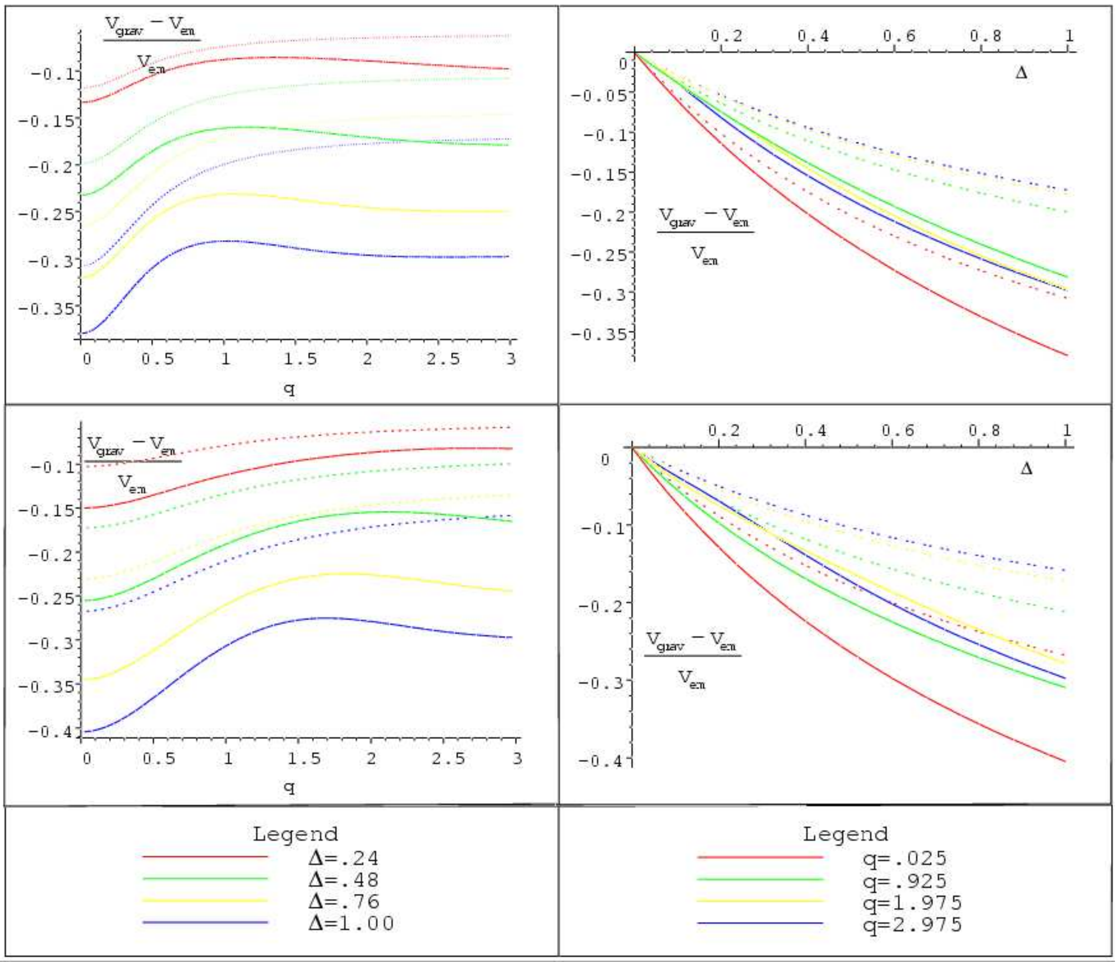

Figure 6: Relative speed difference between the gravity KK modes and hypothetical particles of the same masses, living on the negative tension brane, as a function of q (left) and Δ (right). Upper curves are perturbative result, lower are numerical.

tions of state for the tensions of at least one of the branes, though it is still possible to respect the weak energy condition. One issue we have not explored is the role of a stabilization mechanism such as that of Goldberger and Wise [19] for the size of the extra dimension. One might expect that the bulk is already stabilized once the brane energies and equations of state are fixed, since algebraically the ratio of brane positions r_-/r_+ is determined. However it is possible the radion mass^2 is negative, even though it is not zero. Even if it has the right sign, the distortions of the bulk geometry by a bulk scalar could have an effect on the propagation of gravity. It would also be interesting to know whether pure tension branes could be admitted with the addition of a bulk scalar, as it would be desirable to eliminate the need for an unusual equation of state. Finally, we did not consider how Lorentz violation is manifested in the single brane case, mentioned in section 2.2, although we expect it to be similar to the case of a Planck-brane observer in the two brane model. These subjects could merit further study.

Acknowledgments

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A. Perturbative KK mode solution

The zeroth order in Lorentz-violation KK graviton wave function (which like the zero mode obeys Neumann boundary conditions) is [20]

$$\phi_n^{(0)}(y) = \frac{e^{2k|y|}}{N_n}\left(J_2\left(\frac{m_n e^{k|y|}}{k}\right) - \frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} Y_2\left(\frac{m_n e^{k|y|}}{k}\right)\right), \tag{A.1}$$

where N_n is a normalization constant. The m_i are determined by the boundary conditions; in the limit of a large hierarchy, they satisfy $J_1(m_n e^{ky_-}/k) = 0$.

The wave function correction $\phi_n^{(1)}(y)$ is difficult to compute, but as is familiar from perturbation theory in quantum mechanics, we don't really need it if we are only interested in the correction to the eigenvalue, $\omega_n^{(1)}(q)$. In the quantum mechanical analogy, we require only the matrix element of the perturbation to the hamiltonian. Defining $\psi_n(y) = e^{-2k|y|}\phi_n(y)$, the equation of motion becomes

$$H\psi_n(y) = -\frac{\omega_n^2 e^{2k|y|}}{\hat{h}(y)}\psi_n(y)\,, \tag{A.2}$$

where, to first order in the perturbation,

$$\begin{aligned}
H &= \partial_y(\hat{h}(y)\partial_y) - 4k^2\hat{h}(y) + 2k\,\mathrm{sgn}(y)\,\hat{h}'(y) - q^2 e^{2k|y|} + 4k\hat{h}(y)\delta(y) \\
&= \left[\partial_y^2 - 4k^2 - q^2 e^{2k|y|} + 4k\delta(y)\right] + \\
&\quad + \left[\partial_y(\hat{h}^{(1)}(y)\partial_y) - \left(4k^2 - 4k\delta(y)\right)\hat{h}^{(1)}(y) + 2k\,\mathrm{sgn}(y)\,\hat{h}'^{(1)}(y)\right] \\
&\equiv H^{(0)} + H^{(1)}\,,
\end{aligned} \tag{A.3}$$

with $\hat{h}^{(1)} = \hat{h} - 1$.

We take the inner product with $\psi_n^{(0)}$ and use the Hermicity of $H^{(0)}$ to cancel the terms involving $\psi_n^{(1)}$. This yields

$$\omega_n^{(1)} = \frac{-\langle\psi_n^{(0)}|H^{(1)}|\psi_n^{(0)}\rangle + (\omega_n^{(0)})^2\langle\psi_n^{(0)}|\hat{h}^{(1)}e^{2k|y|}|\psi_n^{(0)}\rangle}{2\omega_n^{(0)}\langle\psi_n^{(0)}|e^{2k|y|}|\psi_n^{(0)}\rangle}\,. \tag{A.4}$$

The inner product in the denominator is unity, with the appropriately normalized wave function. Putting in the explicit form of $H^{(1)}$, the expression becomes

$$\begin{aligned}
\omega_n^{(1)} &= -\frac{1}{\omega_n^{(0)}}\int_0^{y_-}\psi_n^{*(0)}\left(\partial_y(\hat{h}^{(1)}\partial_y) - 4k^2\hat{h}^{(1)} + 2k\hat{h}'^{(1)}\right)\psi_n^{(0)}\,dy + \\
&\quad + \omega_n^{(0)}\int_0^{y_-}\hat{h}^{(1)}e^{2ky}|\psi_n^{(0)}|^2\,dy \\
&\equiv -\frac{1}{\omega_n^{(0)}}A_n + \omega_n^{(0)}B_n\,.
\end{aligned} \tag{A.5}$$

Since $\omega_n^{(0)} = \sqrt{m_n^2 + q^2}$, and $m_n/k \propto e^{-ky_-}$, all the terms are roughly comparable for $q \ll m_n$. But, for large momentum q, only the second integral is important. In that case,

$$\begin{aligned}
\omega_n^{(1)} &\approx q \int_0^{y_-} |\psi_n^{(0)}|^2 \hat{h}^{(1)} e^{2ky}\, dy \\
&= \frac{q}{k^2} \int_+^{y_-} |\psi_n^{(0)}|^2 \left(-\frac{\mu}{r_+^4} e^{6ky} + \frac{Q^2}{r_+^6} e^{8ky} \right) dy \\
&\cong \frac{q}{k^2 N_n^2} \int_+^{y_-} J_2^2 \left(\frac{m_n e^{ky}}{k} \right) \left(-\frac{\mu}{r_+^4} e^{6ky} + \frac{Q^2}{r_+^6} e^{8ky} \right) dy \,, \qquad \text{(A.6)}
\end{aligned}$$

where the last approximation holds in the limit of a large hierarchy. The resulting integral can only be done numerically. However, our analysis is still useful since it tells us that the first correction term to ω_n is proportional to q for large momentum, just as for the zero mode. Moreover we can show how the dispersion relation is expected to change. Squaring ω_n gives

$$\omega_n^2 = (\omega_n^{(0)} + \omega_n^{(1)} + \cdots)^2 \cong (m_n^2 + q^2)(1 + 2B_n - 2A_n) \qquad \text{(A.7)}$$

to first order in the perturbation. Comparing with the usual dispersion relation, one sees that the limiting speed of the nth mode has been modified to

$$c_{\mathrm{grav_n}}^2 = 1 + 2B_n \,, \qquad \text{(A.8)}$$

and that the mass of the mode changes due to the presence of the black hole perturbation:

$$M_n^2 = m_n^2(1 + 2B_n) - 2A_n \,. \qquad \text{(A.9)}$$

In terms of these, the group velocity of the nth mode with momentum q is

$$v_{\mathrm{grav_n}} = \frac{\partial \omega_n}{\partial q} \cong \frac{c_{\mathrm{grav_n}}^2 q}{\sqrt{M_n^2 + c_{\mathrm{grav_n}}^2 q^2}} \,. \qquad \text{(A.10)}$$

As a consistency check, it can be verified that this result reduces to the previous one for the zero mode case where it is possible to do the integrals.

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